Book of abstracts

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Lund University, Campus LTH, Sweden

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Program – Monday

09:30 – 10:00	Registration
10:00 – 10:15	Opening
10:15 – 11:15	Alexander Pushnitski An inverse spectral problem for non-Hermitian Jacobi matrices
11:15 – 11:45	Grzegorz Swiderski On absolute continuity of block Jacobi matrices
12:00 – 14:00	Lunch
14:00 – 15:00	Benjamin Eichinger Necessary and sufficient conditions for universality limits
15:00 – 15:30	Alex Bergman Invariant subspaces for solutions to second order ODE
15:30 – 16:00	Coffee
16:00 – 16:30	Jakob Reiffenstein Determining the order of indeterminate moment problems: a canonical systems approach
16:30 – 17:00	Ayoub Harrat Full moment problem for discrete measures

Program – Tuesday

09:30 – 10:00	Juan Diaz Hahn multiple orthogonal polynomials of type I
10:00 – 11:00	María Ángeles García-Ferrero Exceptional polynomials and how to find them
11:00 – 11:30	Yaozhong Qiu Weyl asymptotics for functional difference operators with potential
11:30 – 12:00	Olena Atlasiuk Approximation of solutions to generic boundary-value problems in Sobolev spaces
12:00 – 14:00	Lunch
14:00 – 17:00	Excursion to Kulturen in Lund

Program – Wednesday

10:00 – 11:00	Maxim Zinchenko Bounds for Chebyshev and extremal polynomials
11:00 – 11:30	Olof Rubin Chebyshev polynomials on polynomial preimage sets
11:30 – 12:00	Teodor Bucht A geometric approach to approximating the limit set of eigenvalues for banded Toeplitz matrices

12:00 – 14:00	Lunch
14:00 – 15:00	Aron Wennman The Christoffel-Darboux kernel without the Christoffel-Darboux formula
15:00 – 15:30	Jiyuan Zhang Stable invariant Hermitian random matrices and the rate of convergence
15:30 – 16:00	Coffee
16:00 – 16:30	Joakim Cronvall Fluctuations near a spectral gap in the random normal matrix model
16:30 – 17:00	Sergey Denisov Szegő condition, scattering, and vibration of Krein strings (on Zoom)

Program – Thursday

10:00 – 11:00	Wafaa Assaad A 3D-Schrödinger operator under magnetic steps with applications in superconductivity
11:00 – 11:30	Matthias Baur Optimization of eigenvalues of the magnetic Dirichlet-Laplacian on planar domains
11:30 – 12:00	Germán Miranda Discrete spectrum of the magnetic Laplacian on almost flat magnetic barriers
12:00 – 14:00	Lunch
14:00 – 15:00	Søren Fournais Counting eigenvalues below the first Landau level for magnetic Schrödinger operators on bounded domains
15:00 – 15:30	Daniel Parra Vogel Continuum limit for a discrete Hodge-Dirac operator on square lattices
15:30 – 16:00	Coffee
16:00 – 16:30	Søren Mikkelsen Sharp semiclassical Weyl law for Schrödinger operators without full regularity
16:30 – 17:00	Volodymyr Mikhailets Spectral analysis of periodic Schrödinger operators with strongly singular potentials
18:30 –	Conference dinner at Restaurant Ishiri

Program – Friday

10:00 – 10:30	Assil Fradi A Riemann-Hilbert approach for Laguerre, Jacobi, and Bessel types matrix orthogonal polynomials: differential operators and Painlevé equations
10:30 – 11:00	Mateusz Piorkowski Biased two periodic Aztec diamond via matrix-valued orthogonal polynomials
11:00 – 12:00	Arno Kuijlaars Matrix-valued orthogonal polynomials and random tilings of a hexagon
12:00 – 12:10	Closing
12:10 – 14:00	Lunch

Abstracts

Plenary talks

A 3D-Schrödinger operator under magnetic steps with applications in superconductivity

Wafaa Assaad, Lebanese International University

In this talk, I will introduce a semiclassical problem in a bounded domain of the space, considering a linear Neumann magnetic Schrödinger operator with a piecewise-constant magnetic field. To motivate studying such a problem, I will give an overview on the potential applications in superconductivity. For a 3D superconductor submitted to certain discontinuous magnetic fields, the localization of the linear operator's ground state and the asymptotics of the corresponding eigenvalue can be used to characterize the intensity of magnetic field at which the superconductor is in a normal state (breakdown of superconductivity). I will outline the variational method used in the study. A new 'effective' Schrödinger operator with a discontinuous magnetic field on the half-space is involved. We will examine this operator. Consequently, I will make precise the localization of the semiclassical ground state near specific points at the discontinuity jump of the magnetic field.

Necessary and sufficient conditions for universality limits

Benjamin Eichinger, TU Wien

In this talk I present necessary and sufficient conditions for universality limits for orthogonal polynomials on the real line. One of our results is that the Christoffel-Darboux kernel has sine kernel asymptotics at a point ξ, with regularly varying scaling, if and only if the orthogonality measure (spectral measure) has a unique tangent measure at ξ and that is the Lebesgue measure. This includes all prior results with absolutely continuous or singular measures. In fact, sine kernel asymptotics is a special case of a more general theory which also includes hard edge universality limits; we show that the Christoffel-Darboux kernel has a regularly varying scaling limit if and only if the orthogonality measure has a unique tangent measure at ξ and that is not the point mass at ξ. In this case the limit kernel is expressible in terms of confluent hypergeometric functions. This talk is based on a joint work in progress with Milivoje Lukić and Harald Woracek.

Counting eigenvalues below the first Landau level for magnetic Schrödinger operators on bounded domains

Søren Fournais, University of Copenhagen

Counting the eigenvalues of elliptic operators is a classical subject with the remarkable Weyl asymptotics being a famous result. These problems are naturally stated in a semiclassical language. In this talk I will focus on a recent work in the 2-dimensional situation in the presence of a constant magnetic field and with Neumann boundary conditions. Suppose we count the eigenvalues up to a level E. In this case there is a transition when E goes from being below to being above the lowest Landau level, that is the lowest eigenvalue for the problem on the entire plane. Below the lowest Landau level the asymptotics is of boundary type, i.e. 1-dimensional in nature. Immediately above, it becomes 2-dimensional. It has been an intriguing open problem to study the behavior exactly at the energy of the lowest Landau level and to determine which dimensionality it belongs to. I will report on new results on this question. This is joint work with Rupert Frank, Magnus Goffeng, Ayman Kachmar, and Mikael Persson Sundqvist.

Exceptional polynomials and how to find them

María Ángeles García-Ferrero, Universitat de Barcelona

Exceptional orthogonal polynomials arise as eigenfunctions of Sturm-Liouville problems and form complete bases in weighted L^2 spaces but, contrary to the classical polynomials of Hermite, Laguerre and Jacobi, their sets of degrees miss finitely many natural numbers. Since 2009, many pages have been written about their construction and their properties, but the book is still unfinished. In this talk, we will summarize the introductory chapters on exceptional orthogonal polynomials and review some of the last lines added to the narrative, mainly regarding the construction of new families which may depend on an arbitrary number of continuous parameters. Joint work with David Gómez-Ullate and Robert Milson.

Matrix-valued orthogonal polynomials and random tilings of a hexagon

Arno Kuijlaars, KU Leuven

The talk is about certain matrix polynomials with non-hermitian orthogonality on a contour in the complex plane. These matrix orthogonal polynomials arise in the analysis of random tilings of planar domains with periodic weightings. I will focus on a particular case of a three-periodic lozenge tiling of a hexagon. The matrix orthogonality is used to obtain the Arctic curves that separate the asymptotic phases of the model, known as the frozen, smooth and rough phases.

An inverse spectral problem for non-Hermitian Jacobi matrices

Alexander Pushnitski, King's College London

The purpose of the talk is to describe an inverse spectral problem for bounded symmetric, but non-Hermitian Jacobi matrices. With each Jacobi matrix J in this class we associate spectral data in a certain natural way. If J is Hermitian, this data reduces to the standard spectral measure of J. The main result is that the map from J to the spectral data of J is a bijection. This includes both injectivity (J is uniquely determined by its spectral data) and the surjectivity (any spectral data corresponds to some J). This is recent joint work with Frantisek Stampach (Prague).

The Christoffel-Darboux kernel without the Christoffel-Darboux formula

Aron Wennman, KTH Stockholm

The Christoffel-Darboux formula is a powerful identity which expresses the reproducing kernel associated to the first n orthogonal polynomials for a measure on the real line solely in terms of the OPs of degrees $n-1$ and n. We focus on the planar setting when the polynomials are orthogonal with respect to weighted area measure, in particular classes of varying exponential weights appearing in random matrix theory. In this context, there is no 3-term recursion and consequently no CD formula. I plan to discuss why we are interested in CD-kernel asymptotics in the above setting and explain some recent developments. In particular, I will present a new PDE-based method which replaces the OPs with a continuously indexed family of orthogonal functions. Under appropriate assumptions on the weight, this gives a strikingly simple global asymptotic formula for the diagonal restriction of the kernel (the "one-point function"). This reports on ongoing joint work with Håkan Hedenmalm (KTH).

Bounds for Chebyshev and extremal polynomials

Maxim Zinchenko, University of New Mexico

In this talk I will give an overview of some of the classical and more recent results for Chebyshev and L^p extremal polynomials. In particular, I will discuss norm estimates for such polynomials on subsets of the real line and the unit circle.

Contributed talks

Approximation of solutions to generic boundary-value problems in Sobolev spaces

Olena Atlasiuk, National Academy of Sciences of Ukraine & Czech Academy of Sciences

We study linear systems of ordinary differential equations on a finite interval with the most general (generic) inhomogeneous boundary conditions in Sobolev spaces. These boundary problems include all known types of classical and numerous nonclassical conditions. The latter may contain derivatives of integer and fractional order, which may exceed the order of the differential equation. We obtained the necessary and sufficient conditions for continuity in a parameter of solutions to the introduced boundary-value problems in the Sobolev spaces. Some applications of these results to the solutions of multipoint boundary-value problems are also presented. The theorem on the approximation of solutions to inhomogeneous generic boundary-value problem by solutions of the multipoint boundary-value problems is proved. The talk is based on joint work with Volodymyr Mikhailets.

Optimization of eigenvalues of the magnetic Dirichlet-Laplacian on planar domains

Matthias Baur, Universität Stuttgart

As a natural extension of the work by Antunes and Freitas on the Dirichlet-Laplacian, we seek to numerically minimize the low eigenvalues as well as sums of low eigenvalues of the planar magnetic Dirichlet-Laplacian with constant magnetic field on bounded, open domains of fixed area. To achieve that, we adapt the Method of Fundamental Solutions to the magnetic Laplacian with constant magnetic field to gain an efficient eigenvalue solver and apply a gradient descent scheme for the shape optimization. For the first eigenvalue we observe that the disc is the minimizer for any magnetic field strength, a result already proven by Erdős in 1996. For higher eigenvalues however we gain a whole zoo of shapes depending on the eigenvalue and the field strength considered. Often the minimizers are connected, but sometimes they appear to be disconnected. Interesting conjectures can be made from the numerical results. Most notable is the observation that for any eigenvalue we observe that for sufficiently strong magnetic field the disc (or something numerically very close to a disc) becomes the minimizer, a simple observation asking heavily for a proof.

Invariant subspaces for solutions to second order ODE

Alex Bergman, Lund University

In 1939 Gelfand posed the problem of determining the invariant subspaces of the Volterra operator,

$$Vf(x) = \int_0^x f(t)dt,$$

on $L^2(0,1)$ (a closed subspace M is called invariant if $VM \subset M$). This attractive problem has been solved by numerous authors. Crucial in certain approaches is that V is the right inverse of the differential operator $D = d/dx$. In this talk we consider generalizations of this result to operators T possessing a left inverse L which has self-adjoint restrictions. In particular we have in mind operators T arising as solutions of second order ODE, e.g. Sturm-Liouville equations or more generally canonical systems. This is based on recent joint work with Alexandru Aleman.

A geometric approach to approximating the limit set of eigenvalues for banded Toeplitz matrices

Teodor Bucht, Lund University

In this talk we discuss a new approach to finding the limit set for banded Toeplitz matrices. The new approach is geometrical and based on the formula $\Lambda(b) = \cap_{\rho\in(0,\infty)} \operatorname{sp} T(b_\rho)$. The outline of our approach is as follows: We find a compact interval of ρ's, $[\rho_l, \rho_h]$ such that $\Lambda(b) = \cap_{\rho\in[\rho_l,\rho_h]} \operatorname{sp} T(b_\rho)$. We then sample ρ's in this interval and for each of the sampled ρ we construct a polygon approximation of $\operatorname{sp} T(b_\rho)$. The intersection of all polygon approximations yields an approximating polygon for $\Lambda(b)$ that converges to $\Lambda(b)$ in the Hausdorff metric. Further, one can slightly expand the polygon approximations for $\operatorname{sp} T(b_\rho)$ to ensure that they contain $\operatorname{sp} T(b_\rho)$. Then, taking the intersection yields an approximating superset of $\Lambda(b)$ which converges to $\Lambda(b)$ in the Hausdorff metric, and is guaranteed to contain $\Lambda(b)$. Finally, we briefly discuss the time complexity and convergence speed for the proposed algorithm.

Fluctuations near a spectral gap in the random normal matrix model

Joakim Cronvall, Lund University

The random normal matrix model is a point process in the complex plane corresponding to eigenvalues of random matrices. The eigenvalues concentrate on a compact set called the droplet. We discuss some new results in the case when the droplet is disconnected. In particular, for radially symmetric models we find the limiting fluctuations of linear statistics and show that they can be expressed as the sum of continuous and discrete Gaussian random variables. We also discuss off-diagonal asymptotics for the associated correlation kernel and prove that it can be expressed as the reproducing kernel of a weighted Hardy space. Joint work with Yacin Ameur and Christophe Charlier.

Szegő condition, scattering, and vibration of Krein strings

Sergey Denisov, University of Wisconsin, Madison

A probability measure μ on the unit circle $\mathbb{T}$ generates a five-diagonal (CMV) matrix $\mathcal{C}$ that is unitarily equivalent to the multiplication by z in $L^2_\mu(\mathbb{T})$. Let $\mathcal{C}_0$ correspond to the Lebesgue measure m. The analysis of polynomials orthogonal with respect to μ shows that the existence of classical wave operators $\lim_{n\to\pm\infty} \mathcal{C}^n\mathcal{C}_0^{-n}$ is equivalent to the measure μ being in the Szegő class, i.e., $\int_{\mathbb{T}} \log \mu' dm > -\infty$. In the talk, we will discuss the analogs of that statement for Krein strings and other models. Based on the joint work with Roman Bessonov from St. Petersburg State University.

Hahn multiple orthogonal polynomials of type I

Juan Diaz, University of Aveiro

The study of multiple orthogonal polynomials and its applications is a very active area of research. However, although many multiple orthogonal polynomials of type II have already been found, there is a lack for analogous expressions for multiple orthogonal polynomials of type I. Hahn is one of those families that lack such expressions. These ones are relevant, among various applications, for being an ascendant of many other families; that can be recovered from them by adequate limits, as indicated in the Askey scheme. Here; explicit expressions for the Hahn multiple polynomials of type I, in terms of Kampé de Fériet hypergeometric series, are given and orthogonality relations are proven. Then; part of the Askey scheme is completed with expressions for the Jacobi-Piñeiro, Meixner, Kravchuk, Laguerre and Charlier type I families.

A Riemann-Hilbert approach for Laguerre, Jacobi, and Bessel types matrix orthogonal polynomials: Differential operators and Painlevé equations

Assil Fradi, University of Aveiro

We examine matrix orthogonal polynomials which are associated with matrices of weights of Laguerre, Jacobi, and Bessel types. These weights are defined using a given matrix Pearson equation. By formulating a Riemann-Hilbert problem, we can obtain first and second order differential equations that the matrix orthogonal polynomials and associated second-kind functions satisfy. Additionally, we can obtain non-Abelian extensions of a discrete Painlevé equation family for the coefficients of the three-term recurrence relation.

Full moment problem for discrete measures

Ayoub Harrat, Université Toulouse III - Paul Sabatier

In this talk, I would like to present an approach allowing to jump from truncated multivariate moment problem to the full one, using idempotent element notion in a Hilbert space. Rather than using a moment matrix approach we utilize here F. Vasilescu method where the central subject is the Riesz functional. This Riesz functional associate the value γ_α to each monomial t^α and satisfies three natural conditions we shall present during the talk, where $(\gamma_\alpha)_{\alpha \in \mathbf{N}^d}$ might be a sequence given by the integral of t^α with respect to a given measure. We also show the crucial role of Λ-multiplicative elements that will be defined during the talk and their relationship with characteristic functions in some L^2 space. The talk is based on a join work with Hamza El-Azhar and Jan Stochel.

Spectral analysis of periodic Schrödinger operators with strongly singular potentials

Volodymyr Mikhailets, National Academy of Sciences of Ukraine & Czech Academy of Sciences

We study the 1D Schrödinger operators with real-valued periodic potentials in negative Sobolev space H^{-1}_{loc}. These operators are bounded below and self-adjoint. They can be defined in the following basic ways: a) as form-sums; b) as quasi-differential operators; c) as limits of operators with smooth coefficients in the norm resolvent sense. The spectra of these operators are absolutely continuous and have band and gap structures. The main results of the talk completely describe the sequences that arise as lengths of spectral gaps with potentials in fractional Sobolev and Hörmander function spaces. The case where the potential is a Radon measure is studied separately. This is joint work with Volodymyr Molyboga.

Sharp semiclassical Weyl law for Schrödinger operators without full regularity

Søren Mikkelsen, University of Bath

In a recent preprint Frank established that the Weyl law for a semiclassical Schrödinger operator acting in $L^2(\mathbb{R}^d)$ holds under minimal assumptions on the potential (arXiv:2202.00323). This result gives the leading order term and an error that is $o(\hbar^{-d})$. By contrast, if the potential is smooth and satisfies some growth conditions, the error is bounded up to a constant by $\hbar^{1-d}$ (Helffer and Robert 1983). This raises the question: At what point, if it exists, is the error no longer $\mathcal{O}(\hbar^{1-d})$? In this talk I will report on results that take steps towards answering this question.

Discrete spectrum of the magnetic Laplacian on almost flat magnetic barriers

Germán Miranda, Lund University

Recently, Bonnaillie-Noël, Fournais, Kachmar and Raymond were able to prove the existence of discrete spectrum of the Neumann magnetic Laplacian on perturbed half-planes. The trial state they used in the proof had a phase which is reminiscent of the one introduced by Correggi and Giacomelli in 2021. We adapt this trial state construction to prove the existence of bound states of a new effective operator involving a magnetic step field on a domain with an almost flat angle. This result emphasizes the fact that even a small non-smoothness of the boundary can cause the appearance of eigenvalues below the essential spectrum. We also give an example where this effective operator arises.

Continuum limit for a discrete Hodge-Dirac operator on square lattices

Daniel Parra Vogel, University of Santiago

We study the continuum limit for Dirac-Hodge operators defined on the n dimensional square lattice $h\mathbb{Z}^n$ as h goes to 0. This result extends to a first order discrete differential operator the known convergence of discrete Schrödinger operators to their continuous counterpart. To be able to define such a discrete analog, we start by defining an alternative framework for a higher-dimensional discrete differential calculus to the standard one defined on simplicial complexes. We then express our operator as a differential operator acting on discrete forms to finally be able to show the limit to the continuous Dirac-Hodge operator.

Biased two periodic Aztec diamond via matrix-valued orthogonal polynomials

Mateusz Piorkowski, KU Leuven

In this talk I will report on our recent progress on the biased two periodic Aztec diamond in a collaboration with Arno Kuijlaars. As shown shown recently by Duits & Kuijlaars '21 the correlation kernels of doubly periodic dimer models can be given in terms of matrix-valued (non-hermitian) orthogonal polynomials. We were able to solve the associated 4×4 Riemann-Hilbert problem exactly, giving us explicit formulae for the orthogonal polynomials and the correlation kernel. Moreover, I will present a new formula for the arctic curve in terms of Jacobi theta functions which proves a recent conjecture by Borodin & Duits '23 stating that such curves are algebraic of degree 8.

Weyl asymptotics for functional difference operators with potential

Yaozhong Qiu, Imperial College London

We start with a review of the coherent state transform, followed by an introduction of functional difference operators and their spectral theory. We then show a generalisation of previous techniques can be used to recover old results and some new ones.

Determining the order of indeterminate moment problems: A canonical systems approach

Jakob Reiffenstein, University of Vienna

This talk is about a new approach to calculating the growth of Nevanlinna matrices associated to indeterminate Hamburger moment problems as well as Jacobi matrices and canonical systems. In the language of canonical systems, rather mild assumptions on the data are sufficient to determine this growth: Certain sequences of coefficients need to be close enough to some regularly varying sequences. Our method gives particular insight into the different behavior for orders less than 1/2 and orders larger than 1/2.

Chebyshev polynomials on polynomial preimage sets

Olof Rubin, Lund University

The Chebyshev polynomial of degree n corresponding to an infinite compact subset E of the complex plane is the unique nth degree monic polynomial which minimises the supremum norm on E. In my talk I will discuss new approaches to determining the asymptotics of such polynomials on sets given as polynomial preimages. In particular, I will show how to relate the Chebyshev polynomials on the complex preimages to weighted Jacobi-Chebyshev polynomials which in turn generalises work due to Lachance, Saff & Varga. Based on joint work together with Jacob Christiansen (LU), Alex Bergman (LU) and Benjamin Eichinger (TU Wien).

On absolute continuity of block Jacobi matrices

Grzegorz Swiderski, University of Wrocław

Khan-Pearson subordinacy theory says that absolute continuity of any scalar Jacobi matrix is equivalent to the statement that the partial ℓ^2 norms of all of its generalized eigenvectors are asymptotically of the same size. In the talk I will discuss a partial generalization of this result to the setup of block Jacobi matrices. This is a joint work with Marcin Moszyński (University of Warsaw).

Stable invariant Hermitian random matrices and the rate of convergence

Jiyuan Zhang, KU Leuven

We consider random matrix ensembles on the Hermitian matrices that are heavy tailed, in particular not all moments exist, and that are invariant under the conjugate action of the unitary group. The latter property entails that the eigenvectors are Haar distributed and, therefore, factorise from the eigenvalue statistics. We prove a classification for stable matrix ensembles of this kind of matrix with the help of the classification of the multivariate stable distributions and the harmonic analysis on symmetric matrix spaces. They can be classified by the stability parameter and the spectral measure, apart from a scaling and a shift. Moreover, we address the question how these ensembles can be generated from the knowledge of the first two quantities. We consider a sum of a specific construction of identically and independently distributed random matrices that are based on Haar distributed unitary matrices and stable random vectors. For this construction, we derive the rate of convergence in the supremum norm and show that this rate is optimal. As a consequence we also give the rate of convergence in the total variation distance.
